Homework Set 4

(sect 1.9 – 2.2)

For questions 1 and 2, assume T is a linear transformation. Find the standard matrix of T.

1. $T: \mathbb{R}^4 \rightarrow \mathbb{R}^2$, $T(\boldsymbol{e_1}) = (1,0)$, $T(\boldsymbol{e_2}) = (5,2)$, $T(\boldsymbol{e_3}) = (2,1)$, and $T(\boldsymbol{e_4}) = (-2,0)$

2. $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $T(\boldsymbol{e_1}) = (1,1,-1)$, $T(\boldsymbol{e_2}) = (0,2,-3)$, and $T(\boldsymbol{e_3}) = (-5,1,0)$

For questions 3 and 4, show that $T(x_1, x_2, x_3) = (0, 5x_2, x_1 - 3x_2, x_2 + x_3)$ is a linear transformation. Note that x_1, x_2, … are not vectors but entries in vectors.

3. Show T is linear using the definition of a linear transformation.

4. Show T is linear by finding a matrix that implements the mapping.

For questions 5 and 6, determine if the linear transformation is (a) one-to-one and (b) onto. Justify each answer. (hint: use the standard matrix of the linear transformation)

5. $T(x_1, x_2, x_3) = (3x_1, 0, x_2 - x_3, 0)$

6. $T(x_1, x_2, x_3, x_4) = 2x_1 - 2x_3 + 3x_4$

For questions 7 through 9, compute each matrix sum or product if it is defined. If an expression is not defined, explain why. Use the matrices defined below.

$$A = \begin{bmatrix} 8 & 0 & 2 \\ -2 & 3 & 6 \end{bmatrix},\ B = \begin{bmatrix} -3 & -1 & 0 \\ 4 & 3 & 5 \end{bmatrix},\ C = \begin{bmatrix} -1 & 2 \\ 4 & 7 \end{bmatrix},\ D = \begin{bmatrix} 2 & 1 \\ 2 & -3 \end{bmatrix}$$

7. $-3A$, $B - 2A$, $5C + D$, AC, CD

8. $5I_2 - D$, $(5I_2)D$

9. Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any will make $AB = BA$?

10. Let $A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$. Verify that $AB = AC$ but $B \neq C$.

Find the inverses of the matrices in questions 12 through 14 if they exist.

11. $\begin{bmatrix} 2 & 7 \\ 4 & 8 \end{bmatrix}$

12. $\begin{bmatrix} 2 & 5 & -1 \\ 3 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

13. $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$

14. Let $A = \begin{bmatrix} 1 & 3 \\ 7 & 13 \end{bmatrix}$, $\boldsymbol{b_1} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$, $\boldsymbol{b_2} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, $\boldsymbol{b_3} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, and $\boldsymbol{b_4} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

a. Find A^{-1}

b. Use what you found in part (a) to solve the 4 equations: $A\boldsymbol{x} = \boldsymbol{b_1}$, $A\boldsymbol{x} = \boldsymbol{b_2}$, $A\boldsymbol{x} = \boldsymbol{b_3}$, and $A\boldsymbol{x} = \boldsymbol{b_4}$

15. Use matrix algebra to show that if A is invertible and D satisfies $AD = I$, then $D = A^{-1}$.